Influence on the entropic force by the virtual degree of freedom on the holographic screen

Qiyuan Pan,[1,2] Bin Wang[1]
[1] *Department of Physics, Fudan University, 200433 Shanghai, China*
[2] *Institute of Physics and Department of Physics, Hunan Normal University, 410081 Changsha, China*

Abstract

We generalize the study of entropic force to a general static spherical spacetime and examine the acceleration, temperature, equation of gravity and the energy associated with the holographic screen in this general background. We show that the virtual degree of freedom on the holographic screen plays a crucial role in interpreting field equations of gravity based on thermodynamical perspective.

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I. INTRODUCTION

The discovery of the thermodynamical properties of black holes inspired deep thinkings on the profound relation between gravity and thermodynamics. A pioneer work on this respect was done by Jacobson who disclosed that the gravitational Einstein equation can be derived from the relation between the horizon area and entropy, together with the Clausius relation $\delta Q = TdS$ [1]. This derivation suggests that the Einstein equation for the spacetime metric has a predisposition to thermodynamic behavior. Jacobson's investigation has been extended to the gravitational theory beyond Einstein gravity, including $f(R)$ gravity [2], the Gauss-Bonnet gravity, the scalar tensor gravity and more general Lovelock gravity [3, 4]. The study has also been generalized to the cosmological context [5], including the general braneworld models [6]. For a review, see [7].

Recently a constructive new idea on the relation between gravity and thermodynamics was proposed by Verlinde [8]. He argued that gravitational interaction is not fundamental and gravity is an emergent entropic force originated from the change of information associated with the positions of bodies of matter. Motivated by Bekenstein's entropy bound, Verlinde postulated that when a test particle with mass m approaches a holographic screen from a distance Δx, the change of entropy on the holographic screen is

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x. \tag{1}$$

The entropic force can arise in the direction of increasing entropy and is proportional to the temperature, $F = T\Delta S/\Delta x$. Adopting the equipartition law of energy

$$E = Mc^2 = \frac{1}{2} N k_B T, \tag{2}$$

where M represents the mass enclosed by the holographic screen and N is the degree of freedom associated with the screen, one can determine the expression for the temperature T. Assuming the total number of bits N on the holographic screen being proportional to the area of the screen A, $N = \frac{Ac^3}{G\hbar}$, Verlinde got the Newton's law of gravity

$$F = G\frac{Mm}{r^2}, \tag{3}$$

where he inserted $A = 4\pi r^2$. Similar observation of the Newton's law of gravity was also obtained by combining the thermodynamical relation $S = E/2T$ with the equipartition law of energy for the horizon degree of freedom [9]. The derivation has been further extended to the relativistic situation and the Einstein equation describing the law of gravity in the relativistic case was obtained [8]. The remarkable idea of the entropic force has been heated discussed recently, see for example [10]-[31].

The assumption that the total number of bits in proportional to the area A of the holographic screen is crucial in Verlinde's derivation. This assumption looks reasonable by taking account of the holographic principle originated from the 'It from bit' picture of Wheeler [32]. However it was argued in [33, 34] that a variant of this picture that takes better account of the symmetries of general relativity is shown to yield corrections to the counting of the degree of freedom that are logarithmic in the area, with a finite, fixed coefficient. If we insert this virtual degree of freedom in Verlinde's derivation, it would be interesting to ask what kind of influence the virtual degree of freedom will bring to the field equations of gravity. In this work we will try to examine this influence.

We will start our discussion with a general class of spherically symmetric and static spacetime and apply Verlinde's approach to investigate the acceleration, temperature and energy on holographic screens and test the entropic force in this general background. In the general spherically symmetric case with a horizon, we will show that the results are consistent with that of Verlinde. This serves the main purpose in this work. Further, we will examine the influence on the Newton gravity and Einstein equation caused by the virtual degree of freedom on the holographic screen.

II. EMERGENT GRAVITY IN GENERAL STATIC SPHERICALLY SYMMETRIC SPACETIME

We consider a general class of spherically symmetric and static four-dimensional spacetime

$$ds^2 = -f(r)dt^2 + \frac{1}{h(r)}dr^2 + R^2(r)(d\theta^2 + \sin^2\theta d\varphi^2), \tag{4}$$

where we relax the condition $f(r) = h(r)$, which is accidentally verified in four dimensions but, in fact, there is no reason for it to continue to be valid in the general scenario. In this spirit, black hole and wormhole solutions as well as star solutions on the brane have been obtained in the last years [35]. For this general class of spherically symmetric line element $f(r)$ and $h(r)$ vanish at the black hole event horizon $r = r_+$, that is $f(r) = f'(r_+)(r - r_+)$ and $h(r) = h'(r_+)(r - r_+)$ as $r \to r_+$, but not necessarily at the same rate. To ensure the metric to be asymptotically flat, we require $f(r)|_{r\to\infty} = h(r)|_{r\to\infty} = 1$. We have adopted units $G = c = \hbar = \kappa_B = 1$. The surface gravity of the event horizon can be determined by

$$\kappa = \frac{1}{2}\sqrt{f'(r_+)h'(r_+)}, \tag{5}$$

where the prime denotes the derivative with respect to r.

For the general static spherically symmetric spacetime, we can express the timelike Killing vector as [8, 30,

31]

$$\xi_\mu = (-f(r), 0, 0, 0). \tag{6}$$

The generalization of the Newton's potential is

$$\phi = \frac{1}{2}\ln(-g^{\mu\nu}\xi_\mu\xi_\nu) = \frac{1}{2}\ln[f(r)], \tag{7}$$

which can be reduced to the ordinary Newtonian potential. This can easily be seen if we look at the simplest case, the Schwarzschild black hole with $f(r) = h(r) = 1 - 2M/r$, we can obtain $\phi = -M/r$ in the large r limit, which is just the ordinary Newtonian potential.

The potential can be used to define the foliation of the space and the holographic screen is at the surface of constant redshift. The acceleration perpendicular to the screen can be expressed as

$$a^\mu = -\nabla^\mu\phi = \left(0, -\frac{h(r)f'(r)}{2f(r)}, 0, 0\right). \tag{8}$$

The local temperature on the screen can now be defined by [8]

$$T = \frac{1}{2\pi}e^\phi\sqrt{\nabla^\mu\phi\nabla_\mu\phi} = \frac{1}{4\pi}\sqrt{\frac{h(r)}{f(r)}}f'(r), \tag{9}$$

where a redshift factor e^ϕ was inserted because the temperature T is measured with respect to the reference point at infinity. This temperature can be considered as Unruh temperature which is closely related to the local acceleration perpendicular to the screen. As Unruh showed, an observer in an accelerated frame can experience this temperature. Furthermore in the spirit of [8] it was argued that this temperature is actually required to cause an acceleration equal to a.

At the black hole event horizon $r = r_+$, we have

$$T|_{r_+} = \frac{1}{4\pi}\sqrt{f'(r_+)h'(r_+)} = \frac{\kappa}{2\pi}, \tag{10}$$

which is just the Hawking temperature T_H relating to the surface gravity κ. When $r \gg r_+$, the metric coefficients can be expanded in power series of $1/r$ in the form

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + O\left(\frac{1}{r^3}\right), \tag{11}$$

$$h(r) = 1 - \frac{2M'}{r} + \frac{Q'^2}{r^2} + O\left(\frac{1}{r^3}\right), \tag{12}$$

where M, M', Q, Q' are expansion parameters. When it is far away from the black hole horizon, we have

$$T|_\infty = 0. \tag{13}$$

The local temperature on the holographic screen is measured with respect to the reference point at infinity. When the holographic screen coincides with the black hole horizon, the temperature on the screen is just the black hole Hawking temperature. When the screen moves away from the horizon, the temperature on the screen changes until it vanishes at spatial infinity.

The force on a particle located very close to the holographic screen can be calculated as

$$F^\mu = T\nabla^\mu S = -me^\phi \nabla^\mu \phi = \left(0, -\frac{mh(r)f'(r)}{2\sqrt{f(r)}}, 0, 0\right), \tag{14}$$

where m is the mass of the test particle. Obviously, with the help of Eq. (8), $F^\mu = me^\phi a^\mu$ is the second law of Newton. This is the gravitational force required to keep a particle at fixed position near the screen as measured from a reference point at infinity [8]. The exponent e^ϕ is the redshift factor, which was inserted in T in Eq. (9), because the temperature T is a local quantity which is measured with respect to the reference point at infinity. The magnitude of this force is

$$F = \sqrt{F^\mu F_\mu} = \frac{m}{2}\sqrt{\frac{h(r)}{f(r)}}f'(r). \tag{15}$$

At the black hole event horizon, it is interesting to note that

$$F|_{r_+} = \frac{1}{2}m\sqrt{f'(r_+)h'(r_+)} = m\kappa. \tag{16}$$

When $r \gg r_+$, using the expansions of the metric coefficients, we arrive at the Newton's law of gravity

$$F = \frac{M_0 m}{r^2}, \tag{17}$$

with the reduced mass

$$M_0 = M\left[1 + (M - M')\frac{1}{r}\right] - \frac{Q^2}{r}. \tag{18}$$

Since we have neglected terms of order $O(1/r^2)$ and higher, Q' does not appear here. The reduced mass M_0 enclosed by the holographic screen in the general static spherical black hole spacetime returns to the black hole mass $M_0 = M$ and $M_0 = M - Q^2/r$ for Schwarzschild black hole and the Reissner-Nordström black hole respectively as observed in [31].

Now we apply the equipartition relation to calculate the energy on the holographic screen. Adopting the assumption $N = A$, we have

$$E = \frac{1}{2}\int_{\mathbb{S}} TdN = \frac{1}{4\pi}\int_{\mathbb{S}} e^\phi \nabla\phi dA = \frac{r^2}{2}\sqrt{\frac{h(r)}{f(r)}}f'(r). \tag{19}$$

When the screen coincides with the black hole horizon, we have

$$E|_{r_+} = \frac{1}{2} r_+^2 \sqrt{f'(r_+)h'(r_+)} = r_+^2 \kappa, \tag{20}$$

which depends on the surface gravity κ. Considering at the horizon $S = A/4 = \pi r_+^2, T|_{r_+} = \kappa/2\pi$, we have $E = 2TS$, which was first obtained in [9] in reinterpreting the law of equipartition from the thermodynamic description at the horizon. When $r \gg r_+$, neglecting terms of order $O(1/r^2)$ and the higher, we get

$$E = M\left[1 + (M - M')\frac{1}{r}\right] - \frac{Q^2}{r}, \tag{21}$$

which is just the reduced mass M_0 related to the black hole in Eq. (18). If we employ the idea in the holographic principle which asserts that the maximum possible number of degrees of freedom is given by a quarter of the area of the screen $S = A/4 = \pi r^2$, we again can have $E = 2TS$ from Eqs. (9) and (19). This shows that the finding at the horizon in [9] keeps on all holographic screens at constant redshifts. On any screen, the law of equipartition is equivalent to the thermodynamic description.

III. VIRTUAL DEGREE OF FREEDOM INFLUENCE ON THE EMERGENT GRAVITY

In the above section, we have discussed the entropic force for the general static spherical spacetime. In getting the entropic force we have adopted the assumption that the number of bits N on the holographic screen is proportional to the area of the screen A. This assumption can be understood from Wheeler's 'It from Bit' picture [32]. Consider that there are p numbers of two dimensional finite 'floating lattice' with size of a Planck area l_p^2 covering the holographic screen. Macroscopically, the classical area of the screen $A \gg l_p^2$. Assume that binary variables (bits) are distributed randomly on the lattice and typically the variables could be with elementary spin 1/2 of $SU(2)$ group [33, 34]. The Hilbert space of quantum states defined by these spin 1/2 variables has a dimensionality $\mathcal{N}(p) = 2^p$ which leads to the number of degree of freedom characterizing the holographic screen $N = p \ln 2$. When $p \gg 1$, the lattices can be taken to approximate the macroscopic screen, $A/l_p^2 = \xi p$. For a choice $\xi = \ln 2$, one has $N = A/G$, where we still keeps $c = \hbar = \kappa_B = 1$.

The generality of the above scenario in counting the degree of freedom of the holographic screen makes it appealing also to the quantum consideration. However in quantum aspect one has to consider the symmetry, which is a crucial aspect of any quantum approach. Considering the elementary variables are spin 1/2 variables under spatial rotations, the symmetry criterion on the physical Hilbert space requires that the Hilbert space consists of states which are compositions of elementary $SU(2)$ doublet states with vanishing total spin. This is the natural choice of the symmetry since in the 'It from Bit' picture, the basic variables are spin 1/2 variables.

This symmetry was also shown arise naturally in the non-perturbative quantum general relativity approach known as quantum geometry [33, 34]. In the large p limit, it was shown that the dimensionality of physical Hilbert space [33, 34]

$$dim\mathcal{H}_{\mathcal{S}} \equiv \mathcal{N}(p) \approx \frac{2^p}{p^{3/2}}, \tag{22}$$

and the number of bits on the holographic screen becomes

$$N = \frac{A}{G}\left[1 - \frac{3}{2}\left(\ln\frac{A}{4G}\right) / \left(\frac{A}{4G}\right)\right], \tag{23}$$

where the overcounting on the degree of freedom has been taken out by considering the symmetry in the quantum approach.

It would be interesting to examine how the virtual degree of freedom on the holographic screen when the quantum aspect is taken into account will influence the equation of gravity. Employing the equipartition law of energy, we can obtain the entropic force due to the change of the virtual information on the screen

$$F = T\frac{\Delta S}{\Delta x} = \frac{GMm}{r^2}\frac{1}{1 - \frac{3}{2}\left(\ln\frac{A}{4G}\right) / \left(\frac{A}{4G}\right)}. \tag{24}$$

Comparing with Eq. (3), we clearly see that the virtual degree of freedom brings the quantum correction to the entropic force. Macroscopically, when we consider $A/G \gg 1$ and $\left(\ln\frac{A}{4G}\right) / \left(\frac{A}{4G}\right) \ll 1$, this quantum correction will be neglected.

In reference [25], the authors assumed that on the surface the information scales proportional to the area of the surface $N = A/G$, and they got the modified force $F = \frac{GMm}{r^2}[1 + 4G\frac{\partial S}{\partial A}]_{A=4\pi r^2}$. Macroscopically, since $\left(\frac{\partial S}{\partial A}\right)_{A=4\pi r^2} \ll 1$, this quantum correction can be neglected. In our study we considered the virtual degree of freedom on the holographic screen (23), which leads to our (24). Although macroscopically in Eq. (24) the quantum correction can be neglected as well, microscopically our result is different from that in [25], since we considered the virtual information on the screen.

We can further consider the influence on the Einstein equation caused by the virtual degree of freedom on the holographic screen. From Eq. (23), it is easy to arrive at

$$dN = \left(\frac{1}{G} - \frac{6}{A}\right) dA. \tag{25}$$

Expressing the energy in terms of the total enclosed mass M and employing the law of equipartition, we have

$$M = \frac{1}{2}\int_{\mathbb{S}} T dN = \frac{1}{4\pi G}\int_{\mathbb{S}}\left(1 - \frac{6G}{A}\right) e^{\phi}\nabla\phi dA. \tag{26}$$

Following the same logic in [8], we can get the integral relation

$$2\int_{\Sigma}(T_{\mu\nu}-\frac{1}{2}Tg_{\mu\nu})N^{\mu}\xi^{\nu}dV=\frac{1}{4\pi G}\int_{\Sigma}R_{\mu\nu}N^{\mu}\xi^{\nu}dV-\frac{3}{2\pi}\int_{\mathbb{S}}\frac{e^{\phi}\nabla\phi}{A}dA. \tag{27}$$

The second term on the right-hand-side is an additional term compared with Eq. (5.37) derived in [8]. This term is caused by the quantum correction to the virtual degree of freedom on the holographic screen which brings a surface correction to the Einstein equation. We can use the simplest Schwarzschild spacetime as an example to further see the role played by this term. For the Schwarzschild spacetime

$$e^{\phi}\nabla\phi=\frac{1}{2}\sqrt{\frac{h(r)}{f(r)}}f'(r)=\frac{MG}{r^2}, \tag{28}$$

we can work out the integration

$$\int_{\mathbb{S}}\frac{e^{\phi}\nabla\phi}{A}dA=4\pi\int_{\mathbb{S}}\frac{GM}{4\pi r^2}\frac{dA}{A}=-\frac{4\pi MG}{A}. \tag{29}$$

Now, the integral relation (27) becomes

$$2\int_{\Sigma}(T_{\mu\nu}-\frac{1}{2}Tg_{\mu\nu})N^{\mu}\xi^{\nu}dV=\frac{1}{4\pi G}\int_{\Sigma}R_{\mu\nu}N^{\mu}\xi^{\nu}dV+2\int_{\Sigma}\epsilon(T_{\mu\nu}-\frac{1}{2}Tg_{\mu\nu})N^{\mu}\xi^{\nu}dV, \tag{30}$$

where we have used $M=\int_{\Sigma}(T_{\mu\nu}-\frac{1}{2}Tg_{\mu\nu})N^{\mu}\xi^{\nu}dV$ and $\epsilon=6G/A\ll 1$ in our discussion. Note that Eq. (30) is always true for arbitrary volume element, so we write in the form the correction to the Einstein equation

$$R_{\mu\nu}=8\pi G(T_{\mu\nu}-\frac{1}{2}Tg_{\mu\nu})+J_{\mu\nu}, \tag{31}$$

where $J_{\mu\nu}=8\pi G\epsilon(\frac{1}{2}Tg_{\mu\nu}-T_{\mu\nu})$ is the very small correction which is from the surface term $\frac{3}{2\pi}\int_{\mathbb{S}}\frac{e^{\phi}\nabla\phi}{A}dA$.

Different from the usual field equations, the term $J_{\mu\nu}$ is a nonlocal effect [21, 28], which is determined by the holographic description of boundary physics in the frame of holography. The additional surface term brings the similarity to the surface term in the Einstein equation discussed in [21, 28], where it was argued arising from the surface term in the action. It was claimed that the correction to the Einstein equation arising from the surface term in the action can be used to explain the universe acceleration [21, 28]. In our case, Eq. (27) was derived in a general static background with a time like Killing vector. It would be interesting to examine the influence due to the virtual degree of freedom on the holographic screen on the gravitation equation in the dynamical spacetimes and see whether the virtual information on the screen can result in the acceleration of the universe.

IV. CONCLUSIONS

In this work we have generalized Verlinde's approach on the entropic force to a general static spherical spacetime. We have studied the acceleration, temperature, gravitational equation and the energy associated

with the holographic screen in the general background. Adopting the assumption that the degree of freedom on the holographic screen $N = A$, we have got the general reduced mass enclosed by the holographic screen. For Schwarzschild and Reissner-Nordström black holes, our general reduced mass returns to that discussed in [31]. We observed that the relation $S = E/2T$ which can be reinterpreted as the law of equipartition not only holds on the black hole horizon as argued in [9], but also on all holographic screens.

In [8], the degree of freedom on the holographic screen was assumed in proportional to the area of the screen. This assumption is crucial in deriving the entropic force. Starting from the Wheeler's 'It from Bit' picture, we have considered the quantum effect in counting the degree of freedom on the screen. We have investigated the influence of the virtual degree of freedom on the equations of gravity. The Newton's law and the Einstein equation are both modified due to the quantum effect in the virtual degree of freedom. This shows that the virtual degree of freedom on the holographic screen plays a crucial role in interpreting field equations of gravity based on thermodynamical perspective. It would be interesting to generalize our study to the dynamical spacetime and examine the effect of the virtual degree of freedom on the holographic screen on cosmic evolution.

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